Compositional and Mechanically Verified Program Analyzers

David Darais
University of Maryland

Let's Design an Analysis

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Property

```
x/0
```

Let's Design an Analysis

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x/0

Program

```
int x y;
void safe_fun(int N) {
 if (N≠0) {x := 0;}
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Value Abstraction

$$\mathbb{Z} \sqsubseteq \{-,0,+\}$$

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Implement

Value Abstraction

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Results

N ∈ {-,0,+}
x ∈ {0,+}
y ∈ {-,0,+}

UNSAFE: {100/N}
UNSAFE: {100/x}

Let's Design an Analysis

Property

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int x y;
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Prove Correct

Value Abstraction

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$$[\![e]\!] \in [\![\mathrm{analyze}(e)]\!]$$

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Flow-insensitive

$$\texttt{results} : \texttt{var} \mapsto \wp(\{-,0,+\})$$

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```
4:     x ∈ {0,+}
4.T: N ∈ {-,+}
5.F: x ∈ {0,+}

N,y ∈ {-,0,+}

UNSAFE: {100/x}
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Flow-sensitive

$$\texttt{results} : \texttt{loc} \mapsto (\texttt{var} \mapsto \wp(\{\texttt{-},\texttt{0},\texttt{+}\}))$$

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N∈{-,+},y∈{-,0,+}
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SAFE
```

Path-sensitive

results : loc ↦ ℘(var ↦ ℘({-,0,+}))

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⟦e⟧ ∈ ⟦analyze(e)⟧

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safe_fun.js ?

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$\mathbb{Z} \sqsubseteq \{-,0,+\}$ ✓

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✗

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Contributions

Orthogonal Components	**Systematic Design**	**Mechanized Proofs**
Galois Transformers [OOPSLA’15]	Abstracting Definitional Interpreters [draft]	Constructive Galois Connections [ICFP’16]

Contributions

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Orthogonal Components

Problem: Isolate path and flow sensitivity in analysis

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Challenge: Path and flow sensitivity are deeply integrated

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State-of-the-art: Redesign from scratch

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State-of-the-art: Redesign from scratch

Our Insight: Monads capture path and flow sensitivity

Galois Transformers

Monadic small-step interpreter

```
type M(t)

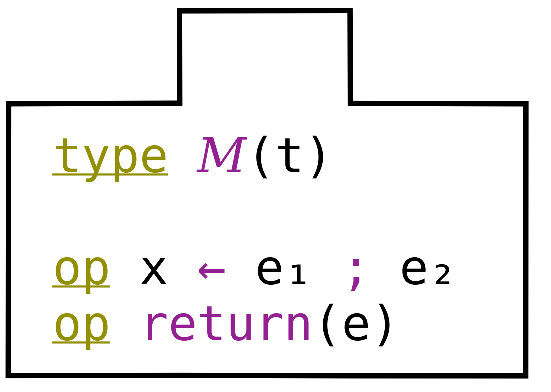
op x ← e₁ ; e₂
op return(e)
```

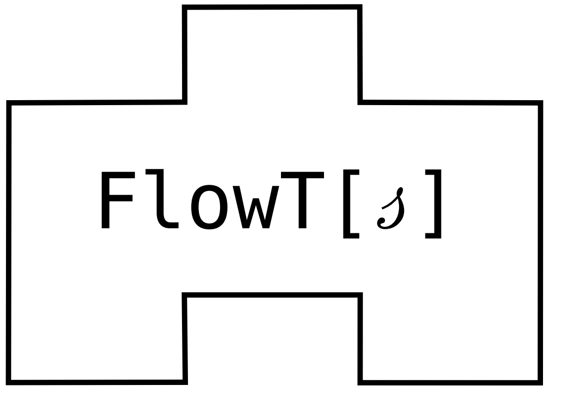
Galois Transformers

Monadic small-step interpreter

+

Monad **Transformers**





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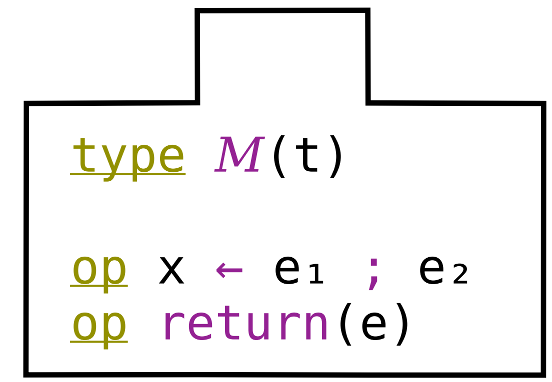

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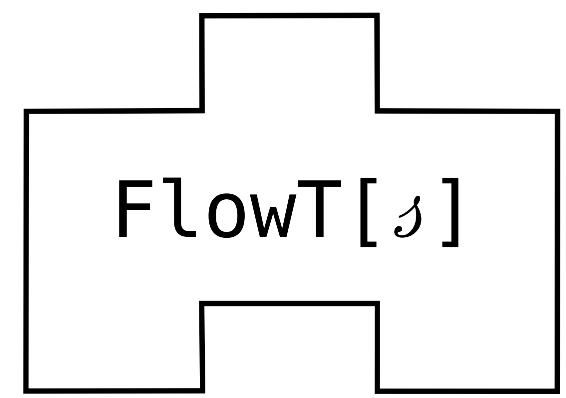

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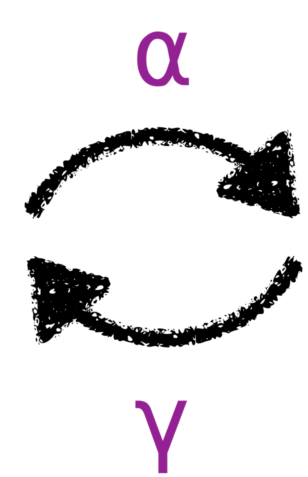
Galois Connections



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✓ Prototype flow insensitive, flow sensitive and path sensitive CFA—no change to code or proof

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- ✗ Not whole story for path-sensitivity refinement
- ✗ Somewhat naive fixpoint iteration strategies

Orthogonal Components

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SAFE
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Prove Correct

$[\![e]\!] \in [\![\text{analyze}(e)]\!]$

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⟦e⟧ ∈ ⟦analyze(e)⟧

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Systematic Design

Problem: Turn interpreters into program analyzers

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Challenge: Interpreters don’t expose reachable configurations

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State-of-the-art: Small-step machines or constraint systems

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Our Insight: Intercept recursion and monad of interpretation

Definitional Abstract Interpreters

Definitional Interpreters ⟦e⟧ : exp → val

Definitional Abstract Interpreters

Definitional Interpreters $[\![e]\!] : \mathtt{exp} \to \mathtt{val}$

\+

Open Recursion $[\![e]\!]^{o} : (\mathtt{exp} \to \mathtt{val}) \to (\mathtt{exp} \to \mathtt{val})$

Definitional Abstract Interpreters

Definitional Interpreters $[\![e]\!]$: exp → val

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Monads (again) $[\![e]\!]^{M}$: exp → M(val)

Definitional Abstract Interpreters

Definitional Interpreters	$[\![e]\!]$: exp → val
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Open Recursion	$[\![e]\!]^{O}$: (exp → val) → (exp → val)
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Custom Fixpoints	Y($[\![e]\!]^{OM}$) vs F($[\![e]\!]^{OM}$)

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- ✗ More complicated meta-theory
- ✗ Monadic, open-recursive interpreters aren't "simple"

Systematic Design

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`x/0`

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`safe_fun.js` ?

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SAFE

Prove Correct

$[\![e]\!] \in [\![analyze(e)]\!]$

Contributions

Orthogonal Components

Galois Transformers [OOPSLA'15]

Systematic Design

Abstracting Definitional Interpreters [draft]

Mechanized Proofs

Constructive Galois Connections [ICFP'16]

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“Calculational Abstract Interpretation” [Cousot99]

Mechanized Proofs

Problem: Calculation, abstraction and mechanization don’t mix

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Challenge: Transition from specifications to algorithms

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State-of-the-art: Avoid Galois connections in mechanizations

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Our Insight: A constructive sub-theory of Galois connections

Calculational Galois Connections

Classical Galois Connections

$$\alpha : \wp(C) \to A$$
$$\gamma : A \to \wp(C)$$

Calculational Galois Connections

Classical Galois Connections

$$\alpha : \wp(C) \rightarrow A$$
$$\gamma : A \rightarrow \wp(C)$$

+

Restricted Form

$$\eta : C \rightarrow A$$
$$\mu : A \rightarrow \wp(C)$$

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Monads (again)

`calculate` $: \wp(A) \rightarrow \wp(A)$

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Monads (again)

$$\texttt{calculate} : \wp(A) \to \wp(A)$$

"has effects" "no effects"

Calculational Galois Connections

✓ First theory to support calculation and extraction

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- ✓ First theory to support calculation and extraction
- ✓ Soundness and completeness, also mechanized
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- ✓ Two case studies: calculational AI and gradual typing
- ✗ Still some reasons not to use Galois connections
- ✗ Calculating abstract interpreters is still very difficult

Mechanized Proofs

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Mechanized Proofs

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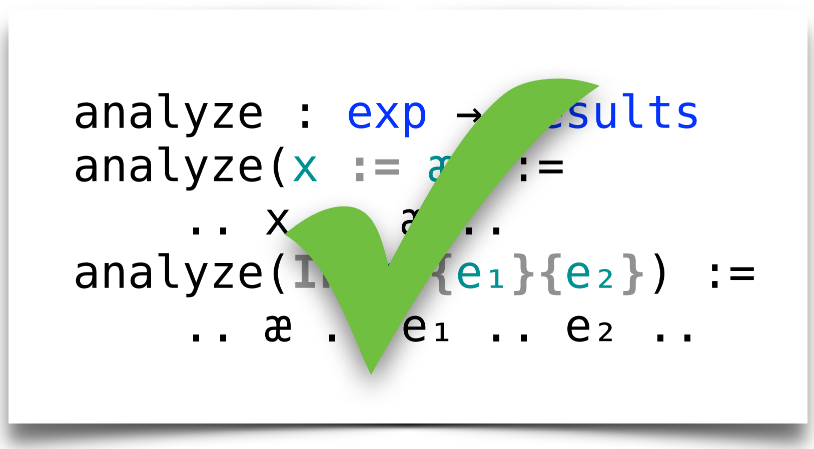

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Program Analysis Design